

**Instructions:** Upload LEGIBLE, COMPLETE solutions to Gradescope before 11:59pm on 15 November 2021.

1. Is the vector field  $\mathbf{v}(x, y)$  conservative? If yes, compute a potential function of  $\mathbf{v}$ .
  - (a)  $\mathbf{v}(x, y) = \langle \exp(x + y^2), y \exp(x + y^2) \rangle$
  - (b)  $\mathbf{v}(x, y) = \langle \sin(x + y) + x \cos(x + y) - y \sin(x), x \cos(x + y) + \cos(x) \rangle$
2. Compute the line integral  $\int_C xye^{yz} dy$  for  $C$  parameterized by  $\mathbf{r}(t) = (t, t^2, t^3)$  on  $0 \leq t \leq 1$ .
3. Use the Fundamental Theorem of Line Integrals to compute the line integral  $\int_C \mathbf{v} \cdot d\mathbf{r}$  for  $\mathbf{v}(x, y, z) = \langle yz, xz, xy + 2z \rangle$  on the line segment from  $(1, 0, -2)$  to  $(4, 6, 3)$ .
4. Use Green's Theorem to compute  $\int_C ye^x dx + 2e^x dy$  along  $C$  the boundary of the rectangle  $[0, 3] \times [0, 4]$ .
5. Use Green's Theorem to compute  $\int_C \mathbf{v} \cdot d\mathbf{r}$  for  $\mathbf{v}(x, y) = \langle y \cos(x) - xy \sin(x), xy + x \cos(x) \rangle$  and  $C$  the clockwise-oriented triangle with vertices  $(0, 0)$ ,  $(0, 3)$ , and  $(1, 0)$ .
6. Compute the curl and divergence of the vector fields below.
  - (a)  $\mathbf{v}(x, y, z) = \langle xy, xy^2z, x + 2y + 3z \rangle$
  - (b)  $\mathbf{v}(x, y, z) = \langle \sin(xyz), xy, \cos(x^2z) \rangle$
7. Is  $\mathbf{v}(x, y, z) = \langle x, y, z \rangle$  the curl of any vector field? Justify completely.